

Nonequilibrium noise correlations in a point contact of helical edge states

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We investigate the noise correlations in a quantum point contact of helical edge states far away from the quantum critical point at finite bias by perturbative calculations. The results reveal the effects of the two-particle scattering processes on the electrical transport, which depend on the Luttinger liquid parameter. Moreover, the Fano factors for the auto- and cross-correlations of the currents are distinct from the ones for tunneling between the chiral edge states in the quantum Hall liquid.

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I. INTRODUCTION

Ever since the discovery of the quantum Hall effect, the study of topological properties of quantum matter continues to be one of the most fruitful subject of researches in condensed matter physics, especially a model of the topological states in the absence of applied magnetic fields was constructed.¹ Recently, on the basis of the earlier works, a new topological state of matter in two dimensions, the quantum spin Hall insulator (QSHI), was theoretically proposed in various systems with time reversal symmetry and spin-orbit interactions.^{2,3} As what happens in the case of the quantum Hall effect, the topological order of the quantum spin Hall state requires the presence of a bulk gap together with gapless edge states.⁴ These edge states propagate in opposite directions for opposite spins, and thus are usually dubbed as the helical liquids.⁵ The stability of the helical liquid against the elastic backscattering is protected by the time-reversal invariance.² Accordingly, the presence of the helical liquid forms a distinctive feature of this new topological state of matter. In this sense, the studies of these helical edge states provide us with important information about the system and help us to identify this new state of matter. This state occurs in HgCdTe quantum well structures,⁶ and there have already been some experimental evidences about the transport properties of helical liquids, which verify the basic features of these unique one-dimensional (1D) system.^{7,8}

In the presence of interactions, we expect that these helical edge states form Luttinger liquids (LLs) in which the spins are associated with the directions of the momenta. Therefore, it is important to have some experimental signatures which show directly the nature of these helical LLs and, in particular, distinguish them from the ordinary LLs. Recently, it was proposed that a quantum point contact in the QSHI can be used as a probe of the helical LL.^{9,10} In Ref. 9, it was noted that the problem of the quantum point contact in a QSHI can be mapped onto the model of a spinful LL with a weak tunneling link. The corresponding LL parameter of the charge mode is

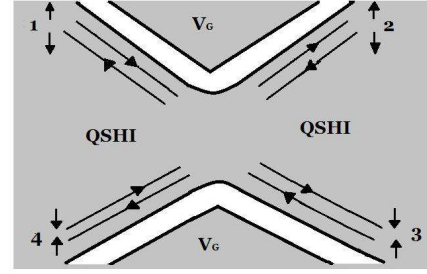


FIG. 1: A quantum point contact in a QSHI. The value of the gate voltage is greater than its critical value so that the point contact is open.

the inverse of that of the spin mode. Therefore, the edge states of the QSHI with a tunneling junction can realize phases which cannot exist for the spin-SU(2) invariant LL with a single impurity.¹¹ It was further shown that there exist a quantum critical point (QCP) which can be tuned by adjusting the value of the gate voltage.¹⁰ Consequently, the low temperature zero-bias conductance can be described by a universal scaling function of the temperature and the gate voltage.

It is important to notice that in determining the phase diagram of the quantum point contact in a QSHI, the two-particle scattering processes, which are naively regarded as less relevant than the single-particle one, play an important role. Thus, it is interesting to see a direct experimental probe of these two-particle scattering processes. One way to achieve this goal is to analyze the noise spectrum of the tunneling junction. In the present work, we shall perform this calculation in the presence of a finite bias V between the top and bottom edges of the point contact when it is open, as shown in Fig. 1, and is far away from the QCP. Hence, a perturbative calculation is reliable. When the point contact is pinched off, the corresponding noise spectrum can be obtained by an appropriate duality transformation.¹⁰ Our main results are shown in Figs. 2–6. Naively, the two-particle scattering processes seem to be more irrelevant than the

single-particle one. But we find that the current and noise spectrum may be dominated by them, depending on the value of the LL parameter,¹³ as shown in Figs. 2–4. To reveal the competition between the single- and two-particle scattering processes clearly, we also calculate the Fano factors for the auto- and cross-correlations. They depend on the LL parameter and the relative strength of the tunneling processes, which can also be viewed as an indirect probe on the possible fractional charges in a helical liquid. This is quite different from the corresponding one for tunneling between chiral edge states in the quantum Hall liquid.^{15,16} Moreover, the Fano factors for the auto- and cross-correlations approach different values in the zero-bias limit, depending on the LL parameter. As we shall discuss in the end of the paper, this result follows from the entanglement of the right and left movers in the final states for different scattering processes.

The rest of the paper is organized as follows. In sec. II, we setup the model to fix our notation. The calculations on the currents and noise spectrum are summarized in Sec. III. The last section is devoted to conclusions and discussions.

II. MODEL

At low energies, the system in Fig. can be described by the Hamiltonian $H = H_0 + \delta H$, where

$$H_0 = \sum_{i=1}^4 \int_0^{+\infty} dx \mathcal{H}_0^{(i)} , \quad (1)$$

with

$$\begin{aligned} \mathcal{H}_0^{(i)} = & iv_F (\psi_{i,\text{in}}^\dagger \partial_x \psi_{i,\text{in}} - \psi_{i,\text{out}}^\dagger \partial_x \psi_{i,\text{out}}) + u_2 J_{i,\text{in}} J_{i,\text{out}} \\ & + \frac{u_4}{2} (J_{i,\text{in}} J_{i,\text{in}} + J_{i,\text{out}} J_{i,\text{out}}) . \end{aligned} \quad (2)$$

Here $\psi_{i,\text{in}}, \psi_{i,\text{out}}$ are a time reversed pair of fermion fields with opposite spin, which propagate toward and away from the junction, v_F is the bare Fermi velocity, and the u_2, u_4 terms are forward scattering. As pointed out in Ref. 9, H_0 can be mapped onto the Hamiltonian of spin-1/2 fermions. To proceed, we define the spin-1/2 fermion fields as

$$\begin{aligned} \psi_{R\uparrow}(x) &= \begin{cases} \psi_{2,\text{out}}(x) & x > 0 \\ \psi_{1,\text{in}}(-x) & x < 0 \end{cases} , \\ \psi_{R\downarrow}(x) &= \begin{cases} \psi_{3,\text{out}}(x) & x > 0 \\ \psi_{4,\text{in}}(-x) & x < 0 \end{cases} , \\ \psi_{L\uparrow}(x) &= \begin{cases} \psi_{3,\text{in}}(x) & x > 0 \\ \psi_{4,\text{out}}(-x) & x < 0 \end{cases} , \\ \psi_{L\downarrow}(x) &= \begin{cases} \psi_{2,\text{in}}(x) & x > 0 \\ \psi_{1,\text{out}}(-x) & x < 0 \end{cases} . \end{aligned} \quad (3)$$

In terms of $\psi_{L\sigma}$ and $\psi_{R\sigma}$, where $\sigma = \uparrow, \downarrow = +, -$, H_0 can be written as

$$H_0 = \int_{-\infty}^{+\infty} dx \mathcal{H}_0 , \quad (4)$$

where

$$\begin{aligned} \mathcal{H}_0 = & \sum_{\sigma} \left[iv_0 (\psi_{L\sigma}^\dagger \partial_x \psi_{L\sigma} - \psi_{R\sigma}^\dagger \partial_x \psi_{R\sigma}) + u_2 J_{L\sigma} J_{R-\sigma} \right] \\ & + \frac{u_4}{2} \sum_{\sigma} (J_{L\sigma} J_{L\sigma} + J_{R\sigma} J_{R\sigma}) . \end{aligned} \quad (5)$$

It is now clear that Eq. (4) is nothing but the Hamiltonian of the spin-1/2 fermions.

Using the bosonization formulas,¹²

$$\begin{aligned} \psi_{L\sigma} &= \frac{1}{\sqrt{2\pi a_0}} \eta_{\sigma} e^{-i\sqrt{4\pi}\phi_{L\sigma}} , \\ \psi_{R\sigma} &= \frac{1}{\sqrt{2\pi a_0}} \eta_{\sigma} e^{i\sqrt{4\pi}\phi_{R\sigma}} , \end{aligned}$$

and defining the bosonic fields

$$\Phi_{\sigma} = \phi_{L\sigma} + \phi_{R\sigma} , \quad \Theta_{\sigma} = \phi_{L\sigma} - \phi_{R\sigma} ,$$

where a_0 is the short-distance cutoff, H_0 becomes

$$H_0 = \sum_{\alpha=c,s} \frac{v_{\alpha}}{2} \int_{-\infty}^{+\infty} dx : \left[K_{\alpha} (\partial_x \Theta_{\alpha})^2 + \frac{1}{K_{\alpha}} (\partial_x \Phi_{\alpha})^2 \right] : , \quad (6)$$

where $K_c = K$, $K_s = 1/K$, $v_c = v = v_s$,

$$\Phi_c = \frac{1}{\sqrt{2}} (\Phi_+ + \Phi_-) , \quad \Phi_s = \frac{1}{\sqrt{2}} (\Phi_+ - \Phi_-) ,$$

and similar expressions for $\Theta_{c,s}$. The Klein factors η_{σ} are usually chosen to satisfy $\eta_+ \eta_- = i$. When spin is conserved at the junction, there are four fixed points.¹¹ These include the perfectly transmitting (CC) limit, in which both charge and spin conduct, the perfectly reflecting (II) limit, in which both charge and spin are insulating, and the mixed fixed points, denoted by CI (IC), in which charge (spin) is perfectly transmitting and spin (charge) is perfectly reflecting. According to the analysis in Refs. 9 and 10, the CC and II phases are separated by a quantum phase transition line by varying the gate voltage. This occurs when $1/2 < K < 2$. This is the region where we shall study.

With the help of Φ_c and Φ_s , δH in the CC limit is given by

$$\begin{aligned} \delta H = & \left[v_e e^{i\sqrt{2\pi}\Phi_c(0)} + \text{H.c.} \right] \cos \left[\sqrt{2\pi}\Phi_s(0) \right] \\ & + \left[v_{\rho} e^{i\sqrt{8\pi}\Phi_c(0)} + \text{H.c.} \right] + v_{\sigma} \cos \left[\sqrt{8\pi}\Phi_s(0) \right] , \end{aligned} \quad (7)$$

In terms of the fermion fields, the various terms in δH can be written as

$$\begin{aligned} v_e : & \psi_{L\sigma}^\dagger \psi_{R\sigma} + \text{H.c.} , \\ v_{\rho} : & \psi_{L\uparrow}^\dagger \psi_{R\uparrow} \psi_{L\downarrow}^\dagger \psi_{R\downarrow} + \text{H.c.} , \\ v_{\sigma} : & \psi_{L\uparrow}^\dagger \psi_{R\uparrow} \psi_{R\downarrow}^\dagger \psi_{L\downarrow} + \text{H.c.} . \end{aligned}$$

Thus, v_e represents the backscattering of a single electron across the point contact, v_ρ denotes the process involving the tunneling of spin (not charge) between the top and bottom edges, and v_σ represents the process involving the tunneling of charge $2e$ between the top and bottom edges. For the weak potential strength, the three terms are irrelevant when $1/2 < K < 2$. In general, higher order terms could also be included. However, those terms are less relevant. It suffices to keep the terms in Eq. (7) to determine the phase diagram. In the following, we shall compute the noise spectrum in the CC limit to study the effects of the two-particle scattering processes.

III. NONEQUILIBRIUM CURRENT AND NOISE

To analyze the transport properties of this system, we apply a voltage bias V between the upper and lower edges of the point contact. In such a case, H_0 becomes

$$\begin{aligned} H_0 &= \sum_{i=1}^4 \int_0^{+\infty} dx \mathcal{H}_0^{(i)} - \sum_{i=1,2} \int_0^{+\infty} dx \mu_+ (J_{i,\text{in}} + J_{i,\text{out}}) \\ &\quad - \sum_{i=3,4} \int_0^{+\infty} dx \mu_- (J_{i,\text{in}} + J_{i,\text{out}}) \\ &= \int_{-\infty}^{+\infty} dx [\mathcal{H}_0 - \mu_+ (J_{R\uparrow} + J_{L\downarrow}) - \mu_- (J_{R\downarrow} + J_{L\uparrow})], \end{aligned}$$

where $\mu_+ - \mu_- = -eV$. (Here we assume that the charge carried by an electron is $-e$.) To proceed, it is convenient to move the dependence on the chemical potentials to δH . This is achieved by the time-dependent gauge transformation: (Throughout the calculations, we set $\hbar = 1$.)

$$\begin{aligned} \psi_{R\uparrow}(\psi_{L\downarrow}) &\rightarrow e^{i\mu_+ t} \psi_{R\uparrow}(\psi_{L\downarrow}), \\ \psi_{R\downarrow}(\psi_{L\uparrow}) &\rightarrow e^{i\mu_- t} \psi_{R\downarrow}(\psi_{L\uparrow}), \end{aligned}$$

leading to $\delta H = \sum_{i=1}^3 \delta H_i$, where

$$\begin{aligned} \delta H_1 &= \left[v_e e^{i\sqrt{2\pi K_c} \tilde{\Phi}_c(t,0)} + \text{H.c.} \right] \\ &\quad \times \cos \left[\sqrt{2\pi K_s} \tilde{\Phi}_s(t,0) - \omega_0 t \right], \\ \delta H_2 &= v_\rho e^{i\sqrt{8\pi K_c} \tilde{\Phi}_c(t,0)} + \text{H.c.}, \\ \delta H_3 &= v_\sigma \cos \left[\sqrt{8\pi K_s} \tilde{\Phi}_s(t,0) - 2\omega_0 t \right]. \end{aligned} \quad (8)$$

Here $\tilde{\Phi}_\alpha = \Phi_\alpha / \sqrt{K_\alpha}$ and $\omega_0 = eV$. The ω_0 dependence of the various terms reflects the numbers of transferred charges involved in the corresponding process.

Let \hat{J}_i denote the particle current operator flowing into lead i . Then, we have

$$\begin{aligned} \hat{J}_1(t, x_1) &= J_{1,\text{in}}(t, -x_1) - J_{1,\text{out}}(t, -x_1) \\ &= J_{R\uparrow}(t, x_1) - J_{L\downarrow}(t, x_1), \\ \hat{J}_2(t, x_2) &= J_{2,\text{in}}(t, x_2) - J_{2,\text{out}}(t, x_2) \\ &= J_{L\downarrow}(t, x_2) - J_{R\uparrow}(t, x_2), \\ \hat{J}_3(t, x_3) &= J_{3,\text{in}}(t, x_3) - J_{3,\text{out}}(t, x_3) \\ &= J_{L\uparrow}(t, x_3) - J_{R\downarrow}(t, x_3), \\ \hat{J}_4(t, x_4) &= J_{4,\text{in}}(t, -x_4) - J_{4,\text{out}}(t, -x_4) \\ &= J_{R\downarrow}(t, x_4) - J_{L\uparrow}(t, x_4), \end{aligned}$$

where $x_1, x_4 < 0$ and $x_2, x_3 > 0$. In terms of the bosonic fields, \hat{J}_i can be written as

$$\begin{aligned} \hat{J}_1 &= \sqrt{\frac{K_s}{2\pi}} \partial_x \tilde{\Phi}_s - \frac{1}{\sqrt{2\pi K_c}} \partial_x \tilde{\Theta}_c = -\hat{J}_2, \\ \hat{J}_3 &= \sqrt{\frac{K_s}{2\pi}} \partial_x \tilde{\Phi}_s + \frac{1}{\sqrt{2\pi K_c}} \partial_x \tilde{\Theta}_c = -\hat{J}_4. \end{aligned} \quad (9)$$

where $\tilde{\Theta}_\alpha = \sqrt{K_\alpha} \Theta_\alpha$. The current flowing into lead i is given by $I_i = -ev_F \langle \hat{J}_i \rangle$. According to our convention, I_i is positive when the current flows out of the lead.

The noise spectrum is defined by

$$S_{ij}(\omega) \equiv e^2 v_F^2 \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \{ \Delta \hat{J}_i(t), \Delta \hat{J}_j(0) \} \rangle, \quad (10)$$

where $\Delta \hat{J}_i = \hat{J}_i - \langle \hat{J}_i \rangle$. We would like to calculate I_i and S_{ij} in terms of the perturbative expansion in the tunneling amplitude v_l ($l = e, \rho, \sigma$) within the Keldysh formalism.¹⁴ We shall see later that $\langle \hat{J}_i \rangle = O(|v_l|^2)$. Thus, to order of $|v_l|^2$, S_{ij} can be written as

$$S_{ij}(\omega) = e^2 v_F^2 \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \{ \hat{J}_i(t), \hat{J}_j(0) \} \rangle.$$

The perturbative calculations of the current and noise spectrum can be straightly performed using the Keldysh functional integral formulation, as that was done in Ref. 15 for the chiral Luttinger liquid of FQHE. To the order of $|v_l|^2$, the currents at zero temperature are given by

$$I_1(t) = -\frac{e}{2} \text{sgn}(\omega_0) \left[|v_e|^2 \text{Re}(\mathcal{A}) |\omega_0|^{K+1/K-1} + |v_\sigma|^2 \text{Re}(\mathcal{B}_s) |2\omega_0|^{4/K-1} \right] = I_2(t) = -I_3(t) = -I_4(t), \quad (11)$$

and the noise spectra at zero temperature are given by

$$S_{ii}(\omega, x) = e^2 \left\{ \frac{K}{\pi} |\omega| + \left[\frac{1-K^2}{2} |v_e|^2 \text{Im}(\mathcal{A}) |\omega_0|^{K+1/K-1} + |v_\sigma|^2 \text{Im}(\mathcal{B}_s) |2\omega_0|^{4/K-1} \right] \sin \left(\frac{2|\omega x|}{v} \right) \right\}$$

$$\begin{aligned}
& + \frac{|v_e|^2}{8}(1-K^2)\left(\mathcal{A}e^{i\frac{2|\omega x|}{v}} + \text{C.c.}\right)\left[|\omega + \omega_0|^{K+1/K-1} + |\omega - \omega_0|^{K+1/K-1}\right] \\
& + \frac{|v_\sigma|^2}{4}\left(\mathcal{B}_s e^{i\frac{2|\omega x|}{v}} + \text{C.c.}\right)\left[|\omega + 2\omega_0|^{4/K-1} + |\omega - 2\omega_0|^{4/K-1}\right] \\
& + |v_e|^2 \text{Re}(\mathcal{A})(|\omega_0| - |\omega|)^{K+1/K-1} \left[\sin^2\left(\frac{\omega x}{v}\right) + K^2 \cos^2\left(\frac{\omega x}{v}\right)\right] \theta(|\omega|) \theta(|\omega_0| - |\omega|) \\
& + 2|v_\sigma|^2 \text{Re}(\mathcal{B}_s)(|2\omega_0| - |\omega|)^{4/K-1} \sin^2\left(\frac{\omega x}{v}\right) \theta(|\omega|) \theta(|2\omega_0| - |\omega|) \\
& - 2K^2 |v_\rho|^2 \left(\mathcal{B}_c e^{i\frac{2|\omega x|}{v}} + \text{C.c.}\right) |\omega|^{4K-1} \Big\}, \tag{12}
\end{aligned}$$

with $i = 1, 2, 3, 4$, and

$$\begin{aligned}
S_{12}(\omega; -x, x) = & -e^2 \left\{ \frac{K|\omega|}{\pi} \cos\left(\frac{2\omega x}{v}\right) - \left[\frac{1+K^2}{2} |v_e|^2 \text{Im}(\mathcal{A}) |\omega_0|^{K+1/K-1} + |v_\sigma|^2 \text{Im}(\mathcal{B}_s) |2\omega_0|^{4/K-1} \right] \sin\left(\frac{2|\omega x|}{v}\right) \right. \\
& - \frac{|v_e|^2}{8}(1+K^2)\left(\mathcal{A}e^{i\frac{2|\omega x|}{v}} + \text{C.c.}\right)\left[|\omega + \omega_0|^{K+1/K-1} + |\omega - \omega_0|^{K+1/K-1}\right] \\
& - \frac{|v_\sigma|^2}{4}\left(\mathcal{B}_s e^{i\frac{2|\omega x|}{v}} + \text{C.c.}\right)\left[|\omega + 2\omega_0|^{4/K-1} + |\omega - 2\omega_0|^{4/K-1}\right] \\
& + |v_e|^2 \text{Re}(\mathcal{A})(|\omega_0| - |\omega|)^{K+1/K-1} \left[\frac{i}{2} \sin\left(\frac{2\omega|x|}{v}\right) + K^2 \cos^2\left(\frac{\omega x}{v}\right) \right] \theta(|\omega|) \theta(|\omega_0| - |\omega|) \\
& + i|v_\sigma|^2 \text{Re}(\mathcal{B}_s)(|2\omega_0| - |\omega|)^{4/K-1} \sin\left(\frac{2\omega|x|}{v}\right) \theta(|\omega|) \theta(|2\omega_0| - |\omega|) \\
& \left. - 2K^2 |v_\rho|^2 \left(\mathcal{B}_c e^{i\frac{2|\omega x|}{v}} + \text{C.c.}\right) |\omega|^{4K-1} \right\}, \tag{13}
\end{aligned}$$

where

$$\begin{aligned}
\text{Re}(\mathcal{A}) &= \frac{\pi a_0^{K+1/K}}{v^{K+1/K} \Gamma(K+1/K)}, \quad \text{Im}(\mathcal{A}) = -\frac{\pi a_0^{K+1/K} \tan[\pi(K+1/K)/2]}{v^{K+1/K} \Gamma(K+1/K)}, \\
\text{Re}(\mathcal{B}_s) &= \frac{\pi a_0^{4/K}}{v^{4/K} \Gamma(4/K)}, \quad \text{Im}(\mathcal{B}_s) = -\frac{\pi a_0^{4/K} \tan(2\pi/K)}{v^{4/K} \Gamma(4/K)},
\end{aligned}$$

and

$$\begin{aligned}
\left(\mathcal{A}e^{i\frac{2|\omega x|}{v}} + \text{C.c.}\right) &= \frac{2\pi a_0^{K+1/K}}{v^{K+1/K} \Gamma(K+1/K)} \left\{ \cos\left(\frac{2|\omega x|}{v}\right) + \tan\left[\frac{\pi}{2}(K+1/K)\right] \sin\left(\frac{2|\omega x|}{v}\right) \right\}, \\
\left(\mathcal{B}_\alpha e^{i\frac{2|\omega x|}{v}} + \text{C.c.}\right) &= \frac{2\pi a_0^{4K_\alpha}}{v^{4K_\alpha} \Gamma(4K_\alpha)} \left[\cos\left(\frac{2|\omega x|}{v}\right) + \tan(2\pi K_\alpha) \sin\left(\frac{2|\omega x|}{v}\right) \right].
\end{aligned}$$

On account of current conservation, the tunneling current I_t is given by $I_t = -(I_1 + I_2) = I_3 + I_4$. This has been verified by directly calculating I_t .

IV. RESULTS AND DISCUSSION

We now discuss our results. The result for the current is shown in Fig. 2. Although what we are considering is the nonequilibrium transport, it is interesting to see that the dependence of each term in Eq. (11) on the bias follows from the scaling dimension of each scattering process. (The scaling dimensions of the v_e , v_ρ , and v_σ terms about the LL fixed point are $\Delta_e = (K+1/K)/2$,

$\Delta_\rho = 2K$, and $\Delta_\sigma = 2/K$, respectively.) We notice that the v_ρ term completely disappears in Eq. (11) because it does not involve net charge transport. Albeit that this term plays an important role in determining the phase diagram, our perturbative calculations shows that its effects on the electrical transport can only be probed through the noise power at finite frequency, as shown in Eqs. (12) and (13).

Next, we consider the noise power. In addition to the singularity at $\omega = 0$, $S_{ij}(\omega)$ also exhibits singularities at $\omega = \omega_0$ and $\omega = 2\omega_0$. They arise from the single-particle and two-particle tunneling, respectively. We expect that these singularities remain intact even by taking into account the non-perturbative effects.¹⁵ Never-

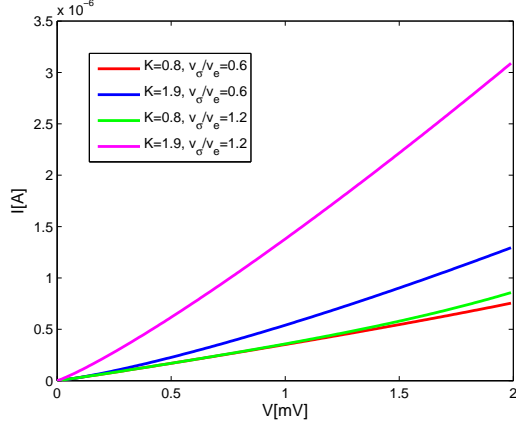


FIG. 2: Dependence of the current $I = |I_i|$ on the bias V . We use the parameters $a_0 = 10^{-7}\text{m}$, $v = 5.5 \times 10^5 \text{ m/s}$, and $|v_e| = \hbar v/a_0$.

theless, in contrast to the tunneling between chiral LLs where only the cross-correlations depend on the position of the probe,¹⁵ both the auto- and the cross-correlations in the present case is sensitive to the position of the probe x . It turns out that their zero-frequency limits are the most robust measurements of fluctuations in the present situation because the resulting expressions in this limit are independent of the position of the probe. The dependence of $S_{11}(0)$ and $S_{12}(0)$ on the bias V is shown in Figs. 3 and 4. We notice that the bias dependence of each term in $S_{11}(0)$ and $S_{12}(0)$ also follows from the scaling dimension of each scattering process. Therefore, among the three scattering terms, only one will dominate the behavior of S_{ij} at low bias, depending on the LL parameter K though the introduction of two-particle scattering will in general enhance the strength of the noise power. It follows from Eqs. (12) and (13) that $S_{ij}(0)$ at low bias are dominated by the single-particle scattering term (the v_e term) at $1/2 < K < \sqrt{3}$, while at $\sqrt{3} < K < 2$ it is the v_σ term that is dominant. A direct consequence of this is that in Figs. 3 and 4, the increase of the magnitude of $S_{ij}(0)$ with $K = 1.9$ at low bias is much larger than the one of $S_{ij}(0)$ with $K = 0.8$ for the same amount of the increment of the ratio $|v_\sigma/v_e|$. A similar situation also occurs for the current, as shown in Fig. 2.

It should be pointed out that, at $\sqrt{3} < K < 2$, the exponents $K + 1/K - 1$ and $4/K - 1$ are numerically quite close to one another. This indicates that both the v_e and v_σ terms will contribute significantly to $S_{ij}(0)$, except when the bias is extremely low. For example, at $K = 1.9$, we must take V to be about 0.01 meV in order that the contribution of the v_σ term is 10 times larger than that of the v_e term, assuming that $|v_\sigma/v_e| = O(1)$. Therefore, a better way to reveal the competition between the single-particle and two-particle scattering processes is to

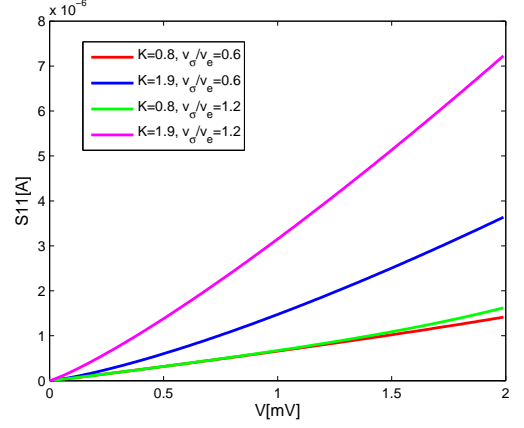


FIG. 3: Dependence of the autocorrelation at zero frequency $S_{11}(0)$ on the bias V . We use the parameters $a_0 = 10^{-7}\text{m}$, $v = 5.5 \times 10^5 \text{ m/s}$, and $|v_e| = \hbar v/a_0$.

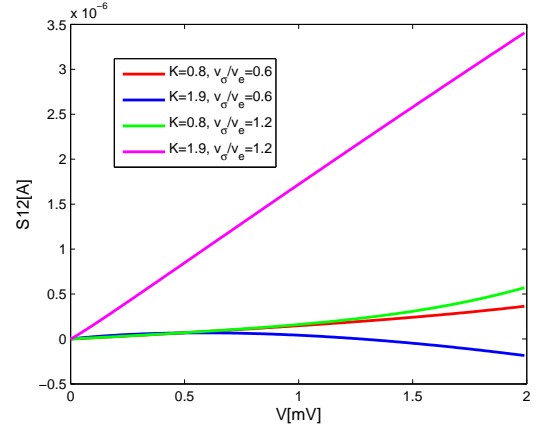


FIG. 4: Dependence of the cross correlation at zero frequency $S_{12}(0)$ on the bias V . We use the parameters $a_0 = 10^{-7}\text{m}$, $v = 5.5 \times 10^5 \text{ m/s}$, and $|v_e| = \hbar v/a_0$.

investigate the Fano factor, which is defined by

$$F_{ij}(V) = \frac{S_{ij}(0)}{2e|I|}, \quad (14)$$

where $S_{ii}(0) = S_{ii}(0, x)$ and $S_{12}(0) = S_{12}(0, -x, x)$. Then, we have

$$\begin{aligned} F_{ii}(V) &= \frac{1}{2} \left[\frac{1 + K^2 + 2\eta|v_\sigma/v_e|^2|\omega_0|^{3/K-K}}{1 + \eta|v_\sigma/v_e|^2|\omega_0|^{3/K-K}} \right], \\ F_{12}(V) &= \frac{1}{2} \left[\frac{1 - K^2 + 2\eta|v_\sigma/v_e|^2|\omega_0|^{3/K-K}}{1 + \eta|v_\sigma/v_e|^2|\omega_0|^{3/K-K}} \right], \end{aligned} \quad (15)$$

where

$$\begin{aligned} \eta &\equiv 2^{4/K-1} \frac{\text{Re}(\mathcal{B}_s)}{\text{Re}(\mathcal{A})} \\ &= 2^{4/K-1} \left(\frac{a_0}{v} \right)^{3/K-K} \frac{\Gamma(K + 1/K)}{\Gamma(4/K)}, \end{aligned}$$

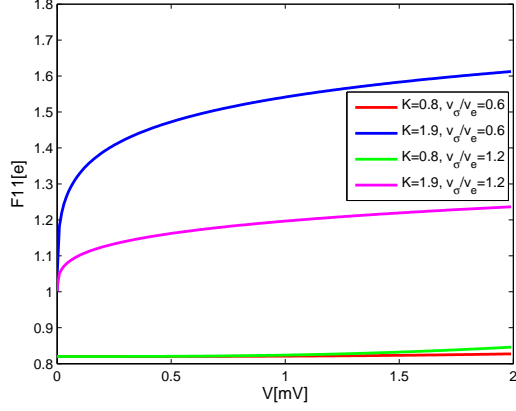


FIG. 5: Dependence of F_{11} on the bias V . We use the parameters $a_0 = 10^{-7}$ m and $v = 5.5 \times 10^5$ m/s.

is a nonuniversal constant. Since the v_ρ dependence completely disappears in the zero-frequency limit, F_{ij} is a function of the single parameter $|v_\sigma/v_e|$. We plot F_{11} and F_{12} as functions of the bias V in Figs. 5 and 6. The effects of the v_e and the v_σ terms are disentangled at the zero-bias limit. By taking $V \rightarrow 0$, we find that

$$F_{ii}(0) = \begin{cases} \frac{1+K^2}{2} & 1/2 < K < \sqrt{3} \\ 1 & \sqrt{3} < K < 2 \end{cases}, \quad (16)$$

and

$$F_{12}(0) = \begin{cases} \frac{1-K^2}{2} & 1/2 < K < \sqrt{3} \\ 1 & \sqrt{3} < K < 2 \end{cases}. \quad (17)$$

In the region where the v_σ term dominates, $F_{ij}(0)$ takes the classical Schottky result, while it depends on the LL parameter K in the region where the single-particle tunneling is dominant. As a byproduct, this may provide a way to measure the value of K for the helical liquid.

Now we try to understand the above results. Since the effects of the v_e and the v_σ terms are disentangled in the limit $V \rightarrow 0$. We first consider this limit. Without loss of generality, we assume that $V > 0$. Then, the v_e term implies a single-electron tunneling from the bottom edge to the top one, whereas the v_σ term implies the simultaneous tunneling of a spin-up electron and a spin-down electron from the bottom edge to the top one. The former produces the following state in the top edge:

$$\begin{aligned} & \sum_{\sigma} \Psi_{\sigma}^{\dagger}(x=0)|O_{LL}\rangle \\ &= \sum_{\sigma} \psi_{R\sigma}^{\dagger}(x=0)|O_{LL}\rangle + \sum_{\sigma} \psi_{L\sigma}^{\dagger}(x=0)|O_{LL}\rangle, \end{aligned}$$

while the state produced by the latter is

$$\psi_{R\uparrow}^{\dagger}(x=0)\psi_{L\downarrow}^{\dagger}(x=0)|O_{LL}\rangle,$$

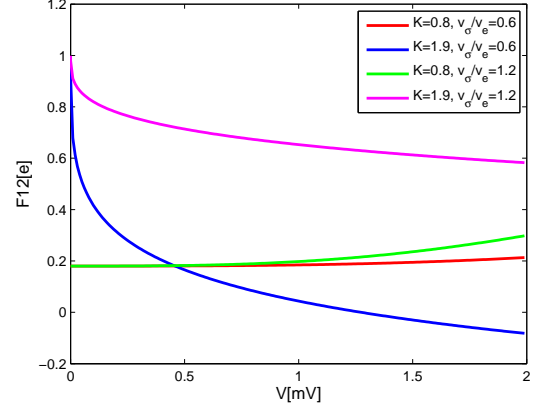


FIG. 6: Dependence of F_{12} on the bias V . We use the parameters $a_0 = 10^{-7}$ m and $v = 5.5 \times 10^5$ m/s.

where $|O_{LL}\rangle$ denotes the ground state of the LL. (Here we follow the idea proposed in Ref. 17.) In the above, the terms with higher scaling dimensions are neglected. To proceed, we define the new chiral bosonic fields

$$\phi_{\alpha l} = \frac{1}{2}(\tilde{\Phi}_{\alpha} + \tilde{\Theta}_{\alpha}), \quad \phi_{\alpha r} = \frac{1}{2}(\tilde{\Phi}_{\alpha} - \tilde{\Theta}_{\alpha}),$$

where $\alpha = c, s$. $\phi_{\alpha l}$ and $\phi_{\alpha r}$ describe the elementary excitations of the spin-1/2 LL propagating with velocity v along the left and the right directions, respectively. In terms of $\phi_{\alpha l}$ and $\phi_{\alpha r}$, we may define the chiral fields carrying a unit of U(1) charge:¹⁸

$$\tilde{\psi}_{cl} = \exp\left[-i\sqrt{\frac{2\pi}{K}}\phi_{cl}\right], \quad \tilde{\psi}_{cr} = \exp\left[i\sqrt{\frac{2\pi}{K}}\phi_{cr}\right]. \quad (18)$$

Then, we have

$$\begin{aligned} & \sum_{\sigma} \Psi_{\sigma}^{\dagger}(x=0)|O_{LL}\rangle \\ &= \left[\tilde{\psi}_{cl}^{\dagger}(x=0)\right]^{Q_-} \left[\tilde{\psi}_{cr}^{\dagger}(x=0)\right]^{Q_+} O_{s1}(x=0)|O_{LL}\rangle \\ &+ \left[\tilde{\psi}_{cl}^{\dagger}(x=0)\right]^{Q_+} \left[\tilde{\psi}_{cr}^{\dagger}(x=0)\right]^{Q_-} O_{s2}(x=0)|O_{LL}\rangle, \end{aligned} \quad (19)$$

and

$$\begin{aligned} & \psi_{R\uparrow}^{\dagger}(x=0)\psi_{L\downarrow}^{\dagger}(x=0)|O_{LL}\rangle \\ &= \tilde{\psi}_{cl}^{\dagger}(x=0)\tilde{\psi}_{cr}^{\dagger}(x=0)O_{s3}(x=0)|O_{LL}\rangle, \end{aligned} \quad (20)$$

where

$$Q_{\pm} = \frac{1 \pm K}{2},$$

and

$$O_{s1} = e^{i\frac{\pi}{16K}(1-K^2)} \sum_{\sigma} \frac{\eta_{\sigma}}{\sqrt{2\pi a_0}} e^{-i\sqrt{\frac{\pi}{2}}\sigma(\Phi_s - \Theta_s)},$$

$$O_{s2} = e^{i\frac{\pi}{16K}(1-K^2)} \sum_{\sigma} \frac{\eta_{\sigma}}{\sqrt{2\pi a_0}} e^{i\sqrt{\frac{\pi}{2}}\sigma(\Phi_s+\Theta_s)},$$

$$O_{s3} = \frac{\eta_{\uparrow}\eta_{\downarrow}}{2\pi a_0} e^{i\frac{(1+K)\pi}{4K}} e^{-i\sqrt{\frac{\pi}{2}}(\Phi_s-\Theta_s)} e^{-i\sqrt{\frac{\pi}{2}}(\Phi_s+\Theta_s)}.$$

Since the operators O_{s1} , O_{s2} , and O_{s3} are charge neutral, we may write Eqs. (19) and (20) as

$$\sum_{\sigma} \Psi_{\sigma}^{\dagger}(x=0)|O_{LL}\rangle \sim |Q_{+}Q_{-}\rangle + |Q_{-}Q_{+}\rangle,$$

$$\psi_{R\uparrow}^{\dagger}(x=0)\psi_{L\downarrow}^{\dagger}(x=0)|O_{LL}\rangle \sim |1,1\rangle,$$

by focusing only on the charge states, where $|Q_l, Q_r\rangle$ denotes the charge state in which the left and the right movers carry charge Q_l and Q_r , respectively. In one spatial dimension, the current fluctuations amount to the measurement of charge fluctuations. Accordingly, we get

$$S_{ii}(0) \propto Q_{+}^2 + Q_{-}^2 = \frac{1+K^2}{2},$$

$$S_{12}(0) \propto 2Q_{+}Q_{-} = \frac{1-K^2}{2}.$$

when the v_e term dominates, while for the v_{σ} term being dominant

$$S_{ij}(0) \propto 1.$$

From the above analysis, we see that the dependence of $F_{ij}(0)$ on the LL parameter K follows from the fact that the final state of the single-particle scattering is an entangled state of $|Q_{+}\rangle$ and $|Q_{-}\rangle$. On the other hand, the classical Schottky result arises from the final state of the two-particle scattering, which is a direct product of the state $|1\rangle$. (This state is not a single-electron state because the left and the right movers carry fractional spins, $-1/K$ and $1/K$ in units of $\hbar/2$, respectively.) At

finite bias, both the v_e and the v_{σ} terms will contribute to the current and the noise power so that the Fano factor depends on the ratio $|v_{\sigma}/v_e|$.

It is interesting to notice that in the case of tunneling between the chiral LLs, the Fano factor takes the classical Schottky result.¹⁵ In the present case, the Fano factor is a function of the LL parameter K even in the absence of the two-particle tunneling. Similar results also occur for tunneling into a nanotube.¹⁷ Hence, this may provide a way to distinguish the spin-1/2 LL and the chiral LL.

Finally, we would like to point out that, as noticed in Ref. 10, there exist a duality relation between the CC and the II limits. Therefore, the noise spectrum in the II limit in the presence of a bias between the left and right edges can be obtained from our results by the interchanging the LL parameters of the charge and spin modes, i.e. $K \leftrightarrow 1/K$.

To summarize, we have studied the current and the noise spectrum of a quantum point contact in the QSHI at finite bias. Special emphasis is put on examining how the single- and the two-particle scattering processes compete with each other. We also point out the difference between the noise spectrums of the helical LL and the chiral LL. These results should provide a useful guide for the experimental identification of the helical liquid, and thus the QSHI.

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